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# Fuzzy Multi-Objective Model for Crop Production Planning with Entropic Disturbance

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**Abstract**

This study incorporates the application of non-linear programming in which there is more than one objective function. In the planning of agriculture, activities play an important part in optimization techniques of the objective values for the composition of mix-type crop production in different land allocations in a particular area. Designing the best production of crops by implementations of land allocation was the main goal of this study. In this article, we considered a multi-objective crop planning model in an area with Shannon's measure of entropy objective function where coefficient parameters of objective functions are taken as trapezoidal fuzzy numbers. Then the said problem is formulated into fuzzy multi objective model and uses a fuzzy decision-making method to solve this problem. Finally, a numerical example has been provided to support the given crop production planning problem.

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**Keywords**

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Multi Objective Model;
λ-Integral Value.

## Introduction

Crop Cultivation is the main sector of INDIA for a major contribution to economic and social development. In agriculture management system plays a significant role in the availability of natural resources in a particular region. Most of the crop production sectors included with favorable agriculture such as compost nature of soil, seasonal factor, labour management, cultivated mix type of crops, maintaining the quantity and quality of crop, fluctuating production etc. can be implemented by successful crop planning and management model. The agriculture production model is usually described as a linear Programming problem. In crop planning and production models, the coefficient of the objective goals/constraints or restrictions parameters are presumed to be a specified certain value. Therefore, there are various diverse circumstances where their exact value remains uncertain that is not be known precisely / accurately and hence uncertainty occurs. So, we need uncertainty-based decision-making methods and here we proposed an uncertainty-based fuzzy programming method. Bellman and Zadeh[1] proposed a fuzzy set theory for decision-making problems in the year 1965. Then 1974 Tanaka *et al.*[2] applied Bellman and Zadeh's concepts of fuzzy

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set theory. In the crop production planning system, where impreciseness and uncertainty perform a vital role in many judgements of the deterministic situations. Sinha *et al.*,[3] contributes various farm planning problems. Sher and Amir[4] applied uncertainty-based fuzzy decision-making methods. Nevo *et al.*[5] developed an integrated crop planning model for agriculture system. Sarker *et al.*[6] formulated a mix crop planning model. Wardlaw and Barnes[7] described optimum assignment of water supply in cultivation area.

Lodwick *et al.*[8] analyses some results by using three different methods for crop production. Itoh and Ishii[9] studied crop planning problems based on possibility measures. Sarker *et al.*[10] introduced nationwide crop planning problems using multicriteria decision-making tools. Toyonaga *et al.*[11] introduced a crop production model to obtain reasonable returns with fuzzy random coefficients. An agriculture planning with of goal programming method developed by Biswas and Pal[12] and Sharma *et al.*[13] Xieting *et al.*[14] formulated a mathematical technique for crop production planning. Optimal crop production with several objective was introduced by Rani and Rao.[15] Raluca Andreea *et al.,*[16] and Kumari *et al*[17] developed different types of crop planning with limited time, resources, budgets etc and reasonably flexible profit at the end of the production. Aminia[18] applied a Fuzzy optimization technique to solve agricultural production model and Sofi *et al.*[19] Mohamed *et al*.,[20-21] Sumpsi *et al.*,[22] develop a muti-objective crop production planning problem for the multi farm. Boyabatli *et al.*,[23] formulated a crop planning in sustainable agriculture and Basumatary and Mitra,[24-25] Mehta and Dwivedi[26] described a model of crop planning in a particular area using fuzzy optimization approach. Zandi *et al.*[27] developed an Analytical Hierarchy Process and failure mode and effects analysis–based agricultural risk management framework using fuzzy TOPSIS.

Various optimization methods are used to solve a non-linear problem. The geometric programming (GP)[28] is one of the best methods for finding the solution a special type of non-linear optimization models. The dual programming of a GP problem can be described as an entropy optimization of mathematical problem. In 1957 Jaynes'[29] introduced an optimization technique called a maximum-entropy principle for finding the distribution of a random system when partial or incomplete data/information is given in the problem Several authors, such as Wilson,[30] Kapur,[31-32] Samanta *et al.*,[33-35] Tsao and Fang,[36] Islam and Roy,[37] used maximum-entropy principle in various scientific fields mainly Operations Research, engineering and technology etc.

The primary objective of this paper is to optimize crop cultivation within a specific area by employing an entropy objective function as a measure of diversification. This approach facilitates the allocation of farmland across multiple crops and seasons, ensuring responsiveness to market demands while accounting for factors such as profit fluctuations, water availability, skilled labour, and climate uncertainty. The entropy-based model is particularly well-suited for farm managers seeking to diversify crop cultivation in a realistic and sustainable manner.

**Formulation of Mathematical Model**

A Mathematical Model for Crop Production (MMCP) is considered under the following notations and assumptions

n = number of different crop cultivation,
m = number of seasons,
$c_{ij}$ = profit coefficient per unit area for a particular crop i in jth season,
$l_{ij}$ = Labour working time for growing crop i in jth season,
$b_{ij}$ = budget of expenditure per unit area to given Compost, pesticide etc. for crop production for a particular crop i in jth season,
B= Total budget for expenditure,
$w_{ij}$ = the water requirement for growing crop i in jth season,
$z_{ij}$ = the required cultivation area for a particular crop i in jth seasons,
A= gross farm land

An MMCP model with minimization of Labour time, the requirement of Water Resources and at the same time the maximization of profit objective with maximum entropic disturbance about the partial/incomplete information (i.e. Shannon's measure of entropy objective function) of farmland distribution under limited farmland and budget for expenditure which can be stated as follows:

$$\text{Maximize } E(z) = -\sum_{j=1}^{m}\sum_{i=1}^{n}\frac{z_{ij}}{A}\ln\frac{z_{ij}}{A} = \frac{1}{A}\left(A\ln A - \sum_{j=1}^{m}\sum_{i=1}^{n} z_{ij}\ln z_{ij}\right)$$
$$\text{Maximize } P(z) = \sum_{j=1}^{m}\sum_{i=1}^{n} c_{ij}z_{ij}$$
$$\text{Minimize } W(z) = \sum_{j=1}^{m}\sum_{i=1}^{n} w_{ij}z_{ij} \qquad ...(1)$$
$$\text{Minimize } L(z) = \sum_{j=1}^{m}\sum_{i=1}^{n} l_{ij}z_{ij}$$
$$\sum_{j=1}^{m}\sum_{i=1}^{n} b_{ij}z_{ij} \le B$$
$$\sum_{j=1}^{m}\sum_{i=1}^{n} z_{ij} = A$$
$$z_{ij} \ge 0,\ i = 1,2,\ldots,n,\ j = 1,2,\ldots,m$$

Now MMCP models can be formulated in crisp and fuzzy environments depending upon the nature of the coefficients of objectives, constraints and goals.

## Generalized Fuzzy Number and λ-integral Value

The MMCP models are solved with consideration of the coefficient parameters of profits, budget, water availability, labour time etc. given in specific certain data that is crisp environment. But in actual real situations, there are many diversified data caused by uncertainty in decisions, insufficient rain/scarcity of water, changed temperature, pesticide effects, market demand etc. Sometimes it becomes impossible to get applicable specific value. Therefore, this not precise/uncertain value is not at all times well characterized by random variables chosen from a distribution of probability. Hence the different types of data of the MMCP problem may be considered to be not exact/approximated(flexible) i.e. Vague (fuzzy) environment.[38]

To illustrate, let profit estimation be vaguely expressed and based on subjective management estimation. It may be imprecisely defined as "Profit is about in the range ( $P_1$, $P_2$ )" i.e. it may have a value within the left spread interval ( $P_1 - \delta^-$, $P_1$ ) and right spread interval ( $P_2$, $P_2 + \delta^+$). Hence this imprecise profit denoted by quadruplet ( $P_1-\delta^-, P_1, P_2, P_2+\delta^+$), may be expressed by the fuzzy set (z, $\mu_{\check{P}}$ (z)) with the membership function $\mu_{\check{P}}$ (z). The $\check{P}$ represents a trapezoidal fuzzy number $\check{P} \equiv ( P_L, P_1, P_2, P_R)$ where $P_L = P_1-\delta^-$ and $P_R=P_2+\delta^+$ and $\delta^-, \delta^+ > 0$.

### Definition

A fuzzy number $\check{P}$ is called Generalized Trapezoidal Fuzzy Number(GTrFN) if $\check{P} \equiv ( P_L, P_1, P_2, P_R, w)$ where $0<w\le 1$ and $P_L$, $P_R$ are the left and right spread of $P_1$, $P_2$ of the range ( $P_1$, $P_2$ ) is a fuzzy set in real line R and its membership value $\mu_{\check{P}}^{w}$ (z) where $\mu_{\check{P}}^{w}$ (z) :R→[0,w] and defined as follows

$$\mu_{\check{P}}^{w}(z) = \begin{cases} \mu_{\check{P}}^{wL}(z) = w\left(\dfrac{z - P_L}{P_1 - P_L}\right) & \text{for } P_L \le z < P_1 \\ w & \text{for } P_1 \le z \le P_2 \\ \mu_{\check{P}}^{wR}(z) = w\left(\dfrac{P_R - z}{P_R - P_2}\right) & \text{for } P_2 < z \le P_R \\ 0 & \text{otherwise} \end{cases}$$

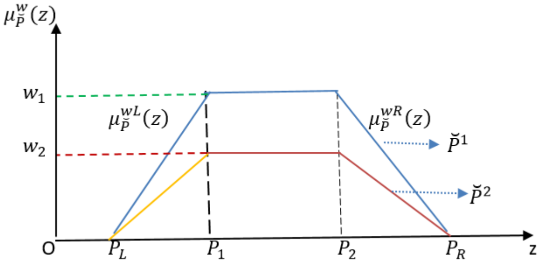


**Fig. 1: Two GTrFN $\check{P}^1 \equiv ( P_L, P_1, P_2, P_R, w_1 )$, $\check{P}^2 \equiv ( P_L, P_1, P_2, P_R, w_2 )$**

Here the left membership function $\mu_{\check{P}}^{wL}$ (z):[$P_L$, $P_1$] →[0,w] is monotonic increasing function and right membership function $\mu_{\check{P}}^{wR}$ (z):[ $P_2$, $P_R$ ]→[0,w] is monotonic decreasing function. More generally, the left GTrFN and right GTrFN can be denoted correspondingly by left $\check{P} \equiv ( P_L, P_1, P_2, P_2, w)$ and right $\check{P} \equiv ( P_1, P_1, P_2, P_R, w)$. The Left GTrFN $\check{P} \equiv ( P_L, P_1, P_2, P_2, w)$ provided $P_L < P_1 \le P_2$ (figure 2) is an appropriate word to define concept 'maximum profit', 'larger', 'high risk' etc. in an interval. Therefore, its membership function:

$$\mu_{\check{P}}^{w}(z) = \begin{cases} w\left(\dfrac{z - P_L}{P_1 - P_L}\right) & \text{for } P_L \le z < P_1 \\ w & \text{for } P_1 \le z \le P_2 \\ 0 & \text{otherwise} \end{cases}$$

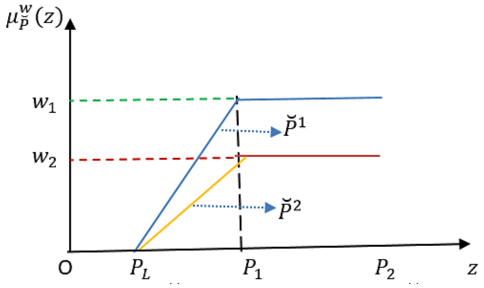


**Fig. 2: Two Left GTrFN $\check{P}^1 \equiv ( P_L, P_1, P_2, P_2, w_1)$, $\check{P}^2 \equiv ( P_L, P_1, P_2, P_2, w_2)$**

Similarly, the right GTrFN (Figure 3.) of $\check{P} \equiv ( P_1, P_1, P_2, P_R, w)$ provided $P_1 \le P_2 < P_R$ is an appropriate word to define concept 'minimum cost', 'little', 'low risk' etc. in an interval. Therefore, its membership function:

$$\mu_{\check{P}}^{w}(z) = \begin{cases} w\left(\frac{P_R - z}{P_R - P_2}\right) & \text{for } P_2 < z \le P_R \\ w & \text{for } P_1 \le z \le P_2 \\ 0 & \text{otherwise} \end{cases}$$

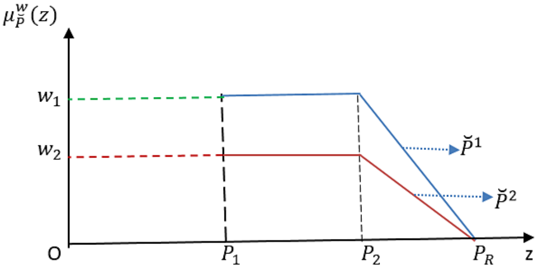


**Fig. 3: Two Right GTrFN $\check{P}^1 \equiv (P_1, P_1, P_2, P_R, w_1)$, $\check{P}^2 \equiv (P_1, P_1, P_2, P_R, w_2)$**

Remark

1: If w =1 then the GTrFN is called TrFN.

2: If $P_1=P_2$ then P̌ is called Generalized Triangular Fuzzy Number.

3: If $P_1=P_2$ and w = 1 then P̌ is called a Triangular Fuzzy Number.

4: If $P_L= P_1=P_2=P_R$ and w = 1 then P̌ is called a real number $P_R$

Let a pre- assigned parameter λ called the degree of satisfaction / optimism where λϵ[0,1]. The Graded Mean Value(GMV) / Total λ Integral Value (TIV) of (P̌) is denoted by $I_{\check{P}}^{w\lambda}$ and defined as $I_{\check{P}}^{w\lambda}=\lambda I_{\check{P}}^{wR}+(1-\lambda) I_{\check{P}}^{wL}$ where $I_{\check{P}}^{w}$ and $I_{\check{P}}^{wL}$ represents right and left TIV of P̌ and are defined as

$$I_{\check{P}}^{wR} = \int_0^1 \left(\mu_{\check{P}}^{wR}\right)^{-1}(v)dv,$$
$$I_{\check{P}}^{wL} = \int_0^1 \left(\mu_{\check{P}}^{wL}\right)^{-1}(v)dv$$

Where $(\mu_{\check{P}}^{wR})^{-1}$ (v) and $(\mu_{\check{P}}^{wL})^{-1}$ (v) represents the inverse of $v=\mu_{\check{P}}^{wR}(z)$ and $v=\mu_{\check{P}}^{wL}(z)$. If $\check{P}\equiv(P_L, P_1, P_2, P_R, w)$ then $(\mu_{\check{P}}^{wR})^{-1}(v)=P_R-(v-w)(P_R-P_2)$ and $(\mu_{\check{P}}^{wL})^{-1}(v)=P_L+(v-w)(P_1-P_L)$.

Therefore, the right and left integral values are $I_{\check{P}}^{wR}=(w/2)(P_2+P_R)$ and $I_{\check{P}}^{wL}=(w/2)(P_L+P_1)$. Hence the TIV of P̌ is $I_{\check{P}}^{w\lambda}=(w/2)[\lambda(P_2+P_R)+(1-\lambda)(P_L+P_1)]$. The right and left integral values are used to reflect the Optimistic and Pessimistic view points of the decision maker respectively. As one can see TIV is the convex combination of both with optimism degree. Therefore, if λ=1, TIV is $I_{\check{P}}^{w1}= I_{\check{P}}^{wR}= (w/2)(P_2+P_R)$ represents Optimistic viewpoint. When λ=0, TIV is $I_{\check{P}}^{w0}=I_{\check{P}}^{wL}=(w/2)(P_L+P_1)$ represents viewpoint of pessimistic situation. Again, if λ=0.5, the TIV is $I\check{P}w_{0.5}= (w/4)[(P_2+P_R)+(P_L+P_1)]=(1/2)(I_{\check{P}}^{wR}+I_{\check{P}}^{wL})$ reflects balanced viewpoint or moderately optimistic viewpoint of the decision makers and it turns out to be the same as defuzzification of the fuzzy number P̌.[39]

### MMCP model with GTrFN

The MMCP model (1) with GTrFN as the coefficients of profit, budget, water availability and labour time of given objective functions which can be represented as follows

$$\begin{aligned} &\text{Maximize } E(z) = -\sum_{j=1}^{m}\sum_{i=1}^{n}\frac{z_{ij}}{A}\ln\frac{z_{ij}}{A} \\ &\text{Maximize } P(z) = \sum_{j=1}^{m}\sum_{i=1}^{n}\check{c}_{ij}z_{ij} \\ &\text{Minimize } W(z) = \sum_{j=1}^{m}\sum_{i=1}^{n}\check{w}_{ij}z_{ij} \\ &\text{Minimize } L(z) = \sum_{j=1}^{m}\sum_{i=1}^{n}\check{l}_{ij}z_{ij} \\ &\quad \sum_{j=1}^{m}\sum_{i=1}^{n}\check{b}_{ij}z_{ij} \le \check{B} \\ &\quad \sum_{j=1}^{m}\sum_{i=1}^{n}z_{ij} = A \\ &\quad z_{ij} \ge 0,\ i = 1,2,\ldots,n,\ j = 1,2,\ldots,m \end{aligned} \qquad \text{...(2)}$$

Where $\check{c}_{ij}$, $\check{b}_{ij}$, $\check{w}_{ij}$, $\check{l}_{ij}$ and B̌ are GTrFN parameters. For left GTrFN $\check{c}_{ij}=(c_{ij}- c_{ij}^{L}, c_{ij}, c_{ij}^{1}, c_{ij}^{1}, w)$ with the tolerance $c_{ij}^{L}$ of the objective function P(z), the TIV of $\check{c}_{ij}$ is $I\check{c}_{ij}^{w\lambda}=(w/2)[2\lambda c_{ij}^{1}+(1-\lambda)(2c_{ij}-c_{ij}^{L})]$. Similarly, right GTrFN of $\check{w}_{ij}$ and $\check{l}_{ij}$ are $\check{w}_{ij}= (w_{ij}, w_{ij}, w_{ij}^{1}, w_{ij}^{1}+w_{ij}^{R}, w)$ and $\check{l}_{ij}=(l_{ij}, l_{ij}, l_{ij}, l_{ij}^{1}+l_{ij}^{R}, w)$ with the tolerances $w_{ij}^{R}$ and $l_{ij}^{R}$ of the objective functions W(z) and L(z) respectively, the TIV are $I_{\check{w}_{ij}}^{w\lambda}=\frac{w}{2}\left[\lambda\left(2w_{ij}^{1}+w_{ij}^{R}\right)+(1-\lambda)2w_{ij}\right]$ and $I_{\check{l}_{ij}}^{w\lambda}=\frac{w}{2}\left[\lambda\left(2l_{ij}^{1}+l_{ij}^{R}\right)+(1-\lambda)2l_{ij}\right]$. For right GTrFN of $\check{b}_{ij}=(b_{ij}, b_{ij}, b_{ij}^{1}, b_{ij}^{1}+b_{ij}^{R}, w)$ and $\check{B}=(B,B,B^{1},B^{1}+B^{R}, w)$ with the tolerances $b_{ij}^{R}$ and $B^{R}$ respectively of $\sum_{j=1}^{m}\sum_{i=1}^{n}\check{b}_{ij}z_{ij} \le \check{B}$, the TIVs of $\check{b}_{ij}$ and B̌ are $I_{\check{b}_{ij}}^{w\lambda}=\frac{w}{2}\left[\lambda\left(2b_{ij}^{1}+b_{ij}^{R}\right)+(1-\lambda)2b_{ij}\right]$ and $I_{\check{B}}^{w\lambda}=\frac{w}{2}\left[\lambda(2B^{1}+B^{R})+(1-\lambda)2B\right]$. Therefore, by using TIV of the fuzzy coefficients, the reformulated form of (2) is

$$\begin{aligned} &\text{Maximize } E(z) = -\sum_{j=1}^{m}\sum_{i=1}^{n}\frac{z_{ij}}{A}\ln\frac{z_{ij}}{A} \\ &\text{Maximize } P(z) = \sum_{j=1}^{m}\sum_{i=1}^{n}I_{\check{c}_{ij}}^{w\lambda}z_{ij} \\ &\text{Minimize } W(z) = \sum_{j=1}^{m}\sum_{i=1}^{n}I_{\check{w}_{ij}}^{w\lambda}z_{ij} \\ &\text{Minimize } L(z) = \sum_{j=1}^{m}\sum_{i=1}^{n}I_{\check{l}_{ij}}^{w\lambda}z_{ij} \\ &\quad \sum_{j=1}^{m}\sum_{i=1}^{n}I_{\check{b}_{ij}}^{w\lambda}z_{ij} \le I_{\check{B}}^{w\lambda} \\ &\quad \sum_{j=1}^{m}\sum_{i=1}^{n}z_{ij} = A \\ &\quad z_{ij} \ge 0,\ i = 1,2,\ldots,n,\ j = 1,2,\ldots,m. \end{aligned} \qquad \text{...(3)}$$

## Material and Methods

### Fuzzy Nonlinear Programming method

A Set of general m linear or non-linear (or both) objective functions and transform them into the Vector Minimization Form as follows

Minimize $O(z)=[O_1(z), O_2(z),\ldots, O_m(z)]$ ...(4)

Subject to $z \in Z=\{ z: g_k (z)\leq d_k, k=1,2,\ldots,n\}$

The solution of (4) by using Zimmerman[40] Max-Min operator method as following

Find z ...(5)

so as to satisfy $O_r(z) \lesssim L_r$, $r = 1,2,\ldots,m$.

$z \in Z$

Where $L_r$ are corresponding goals of the objective function $O_r(z)$ which are to be minimized and considered these objective functions of (4) as fuzzy constraints. The notation $\mu_r(O_r(z))$ represents the membership functions corresponding to $O_r(z)$. This membership functions are strictly monotonic decreasing function and to calculate payoff table by using Ideal Solution of objective functions and constraints. Hence a set of feasible solution is represented by its membership function is

$\mu_{\tilde{D}}(z)$=Minimize$\mu_{\tilde{D}}\{\mu_1(O_1(Z)),\mu_2(O_2(Z))\ldots\mu_m(O_m(Z))\}$

Now a decision maker can be used a fuzzy optimization method and hence taking maximum level of $\mu_{\tilde{D}}$ (z) the feasible solution set, so

Maximize $\mu_{\tilde{D}}$ (z) ...(6)
subject to $z \in Z$

If $\alpha$ ( $0\leq \alpha\leq 1$ ) be the all compromise satisfactory level then the problem (6) can be reduced to

Maximize $\alpha$ ... (7)

such that $\mu_r (O_r (Z)) \geq \alpha$, for $r = 1, 2, \ldots, m$ .
$z \in Z$.
To solve (7) we get Pareto Optimal Solution.

**Definition : Optimal Solution (Complete)**
We can said z* as an optimal solution (Complete) to the problem (4) iff for all $z \in Z$ and there exists $z^* \in Z$ such that $O_k(z^*) \leq O_k(z)$, for $k = 1, 2, \ldots, m$.

The Nonlinear problem (4) does not always exist an optimal solution(complete) because many times arises conflict nature of the problems objective functions. Therefore, arises another concept of optimal solution called Optimal Solution (Pareto) as follows:

**Definition: Optimal Solution (Pareto)**
We can said z* as an Optimal Solution (Pareto) to the problem (4) iff there does not exist another $z \in Z$ such that for all $k =1, 2, \ldots, m$ , $O_k (z^*) \leq O_k(z)$ and for at least one j, $j \in < 1,2,\ldots,m >$, $O_j(z) \neq O_j(z^*)$.

**Solution Procedure of MMCP by using above solution Procedure**
The vector minimization form of problem (3) is

$$\begin{aligned}
&\text{Minimize } [-E(z)] = \sum_{j=1}^{m}\sum_{i=1}^{n}\frac{z_{ij}}{A}\ln\frac{z_{ij}}{A}\\
&\text{Minimize } [-P(z)] = -\sum_{j=1}^{m}\sum_{i=1}^{n} I_{\tilde{c}_{ij}}^{w\lambda} z_{ij}\\
&\text{Minimize } W(z) = \sum_{j=1}^{m}\sum_{i=1}^{n} I_{\tilde{w}_{ij}}^{w\lambda} z_{ij}\\
&\text{Minimize } L(z) = \sum_{j=1}^{m}\sum_{i=1}^{n} I_{\tilde{l}_{ij}}^{w\lambda} z_{ij} \qquad \ldots (8)\\
&\sum_{j=1}^{m}\sum_{i=1}^{n} I_{\tilde{b}_{ij}}^{w\lambda} z_{ij} \leq I_{\tilde{B}}^{w\lambda}\\
&\sum_{j=1}^{m}\sum_{i=1}^{n} z_{ij} = A\\
&z_{ij} \geq 0,\ i = 1,2,\ldots,n,\ j = 1,2,\ldots,m
\end{aligned}$$

Now to solve the above problem (8), we are used in the above solution procedure.
Here the membership functions $\mu_{-E}(-E(Z))$, $\mu_{-P}(-P(Z))$, $\mu_W(W(Z))$ and $\mu_L(L(Z))$ for the objective functions $[-E(z)]$, $[-P(z)]$, $W(z)$ and $L(z)$ respectively are defined as follows:

$$\mu_{-E}(-E(z)) = \begin{cases} 1 & \text{for } -E(z) \leq -U_E \\ \frac{-L_E-(-E(z))}{-L_E(-U_E)} & \text{for } -U_E \leq -E(z) \leq -L_E \\ 0 & \text{for } -E(z) \geq -L_E \end{cases}$$

$$\mu_{-P}(-P(z)) = \begin{cases} 1 & \text{for } -P(z) \leq -U_P \\ \frac{-L_P-(-P(z))}{-L_P(-U_P)} & \text{for } -U_P \leq -P(z) \leq -L_P \\ 0 & \text{for } -P(z) \geq -L_P \end{cases}$$

$$\mu_W(W(z)) = \begin{cases} 1 & \text{for } W(z) \leq L_W \\ \frac{U_W-W(z)}{U_W-L_W} & \text{for } L_W \leq W(z) \leq U_W \\ 0 & \text{for } W(z) \geq U_W \end{cases}$$

$$\mu_L(L(z)) = \begin{cases} 1 & \text{for } L(z) \leq L_L \\ \frac{U_L-L(z)}{U_L-L_L} & \text{for } L_L \leq L(z) \leq U_L \\ 0 & \text{for } L(z) \geq U_L \end{cases}$$

where the $U_E$, $U_P$, $U_W$, $U_L$ and $L_E$, $L_P$, $L_W$, $L_L$ are the upper and lower bounds of the objective functions $[-E(z)]$, $[-P(z)]$, $W(z)$ and $L(z)$ and its evaluated by estimated pay off table.
If $\beta$ ( $0\leq \beta\leq 1$ ) be the all compromise satisfactory level then the above problem (8) is calculated as the following form:

Maximize β ...(9)

subject to

$$E(z) \geq L_E + \beta(U_E - L_E)$$
$$P(z) \geq L_P + \beta(U_P - L_P)$$
$$W(z) \leq U_W - \beta(U_W - L_W)$$
$$L(z) \leq U_L - \beta(U_L - L_L)$$
$$\sum_{j=1}^{m} \sum_{i=1}^{n} I_{\breve{b}_{ij}}^{w\lambda} z_{ij} \leq I_{\breve{B}}^{w\lambda}$$
$$\sum_{j=1}^{m} \sum_{i=1}^{n} z_{ij} = A$$
$$z_{ij} \geq 0,\ i = 1,2,\ldots,n,\ j = 1,2,\ldots,m \quad and\ \beta \in [0,1].$$

**Numerical Example**

Suppose an agriculture manager assigns their limited farmland areas nearly 45 hectares and he want to grow three types of crops say, Paddy, Brinjal and Red Chili in three different seasons namely Summer, Rainy and Winter. About his past experience, he stated that the total budget of expenditure for cultivation is approximately in the range(interval) Indian Rupees 102 lakhs to 105 lakhs, The following table represents the range of budget for expenditure ( in lakhs), work time (in day=24 hours) for labour and water availability( in hector-inches) for the crop production per unit hector area and his past three year ( three seasons) experience of profit coefficients range (in lakh) to grow five different crop are given :

**Table 1: Season wise Crop information**

| Season (j) | Crop Name (i) | Profit coefficients ( lakhs) | Water availability (hector-inches) | Work time for Labour (days) | Budget for expenditure (lakhs) |
|---|---|---|---|---|---|
| Summer Season | Paddy ($z_{11}$) | 1.10 –1.15 | 19.4 –19.5 | 2.40 –2.50 | 3.44 – 3.50 |
| | Brinjal($z_{21}$) | 1.00 – 1.05 | 17.2 –17.4 | 1.75 –1.80 | 2.42 – 2.50 |
| | Red Chili($z_{31}$) | 1.20 –1.25 | 18.3 –18.5 | 1.88 –1.90 | 2.64 – 2.70 |
| Rainy Season | Paddy ($z_{12}$) | 1.40 –1.44 | 24.8 –25.0 | 1.95 –2.00 | 2.98 – 3.00 |
| | Brinjal($z_{22}$) | 1.10 – 1.16 | 21.7 –22.0 | 1.68 –1.70 | 2.38 – 2.40 |
| | Red Chili($z_{32}$) | 1.30 – 1.36 | 19.2 – 20.0 | 1.66 –1.70 | 2.34 – 2.40 |
| Winter Season | Paddy ($z_{13}$) | 1.20 – 1.24 | 21.4 –22.0 | 2.30 –2.40 | 2.72 – 2.80 |
| | Brinjal($z_{23}$) | 1.20 – 1.29 | 18.2 –18.5 | 1.67 –1.70 | 2.19 – 2.20 |
| | Red Chili($z_{33}$) | 1.40 – 1.46 | 19.3 –19.5 | 1.94 –2.00 | 2.12 – 2.20 |

The Manager assigned his farmland areas i.e. $z_{ij}$= cultivation land area of the corp i ( i=1,2,3) in jth (j=1,2,3) season, therefore the mathematical formulation of imprecise / flexible data of different coefficients and goals, so the model are as follows:

Maximize E(z) = $-\left(\frac{z_{11}}{45}\ln\frac{z_{11}}{45} + \frac{z_{21}}{45}\ln\frac{z_{21}}{45} + \frac{z_{31}}{45}\ln\frac{z_{31}}{45} + \frac{z_{12}}{45}\ln\frac{z_{12}}{45} + \frac{z_{22}}{45}\ln\frac{z_{22}}{45} + \frac{z_{32}}{45}\ln\frac{z_{32}}{45} + \frac{z_{13}}{45}\ln\frac{z_{13}}{45} + \frac{z_{23}}{45}\ln\frac{z_{23}}{45} + \frac{z_{33}}{45}\ln\frac{z_{33}}{45}\right.$

Maximize P(z) = $\breve{(}1.10-1.15)\, z_{11} + \breve{(}1.00-1.05)\, z_{21} + \breve{(}1.20-1.25)\, z_{31} + \breve{(}1.40-1.44) z_{12} + \breve{(}1.10-1.16)\, z_{22} + \breve{(}1.30-1.36)\, z_{32} + \breve{(}1.20-1.24)\, z_{13} + \breve{(}1.20-1.29)\, z_{23} + \breve{(}1.40-1.46)\, z_{33}$

Minimize W(z) = $\breve{(}19.4-19.5)\, z_{11} + \breve{(}17.2-17.4)\, z_{21} + \breve{(}18.3-18.5)\, z_{31} + \breve{(}24.8-25.0) z_{12} + \breve{(}21.7-22.0)\, z_{22} + \breve{(}19.2-20.0)\, z_{32} + \breve{(}21.4-22.0)\, z_{13} + \breve{(}18.2-18.5)\, z_{23} + \breve{(}19.3-19.5)\, z_{33}$

Maximize L(z) = $\breve{(}2.40-2.50)\, z_{11} + \breve{(}1.75-1.80)\, z_{21} + \breve{(}1.88-1.90)\, z_{31} + \breve{(}1.95-2.00) z_{12} + \breve{(}1.68-1.70)\, z_{22} + \breve{(}1.66-1.70)\, z_{32} + \breve{(}2.30-2.40)\, z_{13} + \breve{(}1.67-1.70)\, z_{23} + \breve{(}1.94-2.00)\, z_{33}$

$\breve{(}3.44-3.50)\, z_{11} + \breve{(}2.42-2.50)\, z_{21} + \breve{(}2.64-2.70)\, z_{31} + \breve{(}2.98-3.00)\, z_{12} + \breve{(}2.38-2.40)\, z_{22} + \breve{(}2.34-2.40)\, z_{32} + \breve{(}2.72-2.80)\, z_{13} + \breve{(}2.19-2.20)\, z_{23} + \breve{(}2.12-2.20)\, z_{33} \leq \breve{(}102-105)$

$z_{11} + z_{21} + z_{31} + z_{12} + z_{22} + z_{32} + z_{13} + z_{23} + z_{33} = 45$

$z_{ij} \geq 0, i = 1,2,3, j = 1,2,3.$

Where fuzzy objective coefficient of P(z) is $\breve{(}1.10-1.15)=(1.0,1.10,1.15,1.15;0.9)$, $\breve{(}1.00-1.05)=(0.9,1.00,1.05,1.05;0.8)$, $\breve{(}1.20-1.25)=(1.10,1.20,1.25,1.25;0.7)$, $\breve{(}1.40-1.44)=(1.35,1.40,1.44,1.44;0.8)$ $\breve{(}1.10-1.16)=(1.0,1.10,1.16,1.16;0.7)$, $\breve{(}1.30-1.36)=(1.20,1.30,1.36,1.36;0.9)$, $\breve{(}1.20-1.24)=(1.15,1.20,1.24,1.24;0.9)$, $\breve{(}1.20-1.29)=(1.10,1.20,1,29,1,29;0.7)$ $\breve{(}1.40-1.46)=(1.30,1.40,1.46,1.46;0.8)$

Similarly fuzzy objective coefficient of W(z) is $\breve{(}19.4-19.5)=(19.4,19.4,19.5,19.6;0.8)$, $\breve{(}17.2-17.4)=(17.2,17.2,17.4,17.6;0.9)$, $\breve{(}18.3-18.5)=(18.3,18.3,18.5,18.7;0.7)$, $\breve{(}24.8-25.0)=(24.8,24.8,25.0,25.2;0.8)$ $\breve{(}21.7-22.0)=(21.7,21.7,22.0,22.3;0.7)$, $\breve{(}19.2-20.0)=(19.2,19.2,20.0,20.8;0.9)$, $\breve{(}21.4-22.0)=(21.4,21.4,22.0,22.6;0.7)$, $\breve{(}18.2-18.5)=(18.2,18.2,18.5,18.8;0.8)$, $\breve{(}19.3-19.5)=(19.3,19.3,19.5,19.7;0.9)$

and fuzzy objective coefficient of L(z) is ˜(2.40 -2.50)=(2.40,2.40,2.50,2.60;0.7), ˜(1.75 -1.80)=(1.75,1.75,1.80,1.85;0.8), ˜(1.88 -1.90)=(1.88,1.88,1.90,1.92;0.7), ˜(1.95 -2.00)=(1.95,1.95,2.00,2.05;0.9), ˜(1.68-1.70)=(1.68,1.68,1.70,1.70,1.72;0.8), ˜(1.66 -1.7)=(1.66,1.66,1.70,1.74;0.6), ˜(2.30 -2.40)= (2.30,2.30,2.40,2.50;0.8), ˜(1.67-1.70)=(1.67,1.67,1.70,1.73;0.9), ˜(1.94 -2.00)=(1.94,1.94,2.00,2.06;0.7)

Fuzzy Coefficients and goals of Budget constraints is ˜(3.44 - 3.50)=(3.44,3.44,3,50,3.56;0.7),( ˜2.42 - 2.50)=(2.42,2.42,2.50,2.58;0.8), ( ˜2.64 - 2.70)=(2.64,2.64,2.70,2.76;0.9),( ˜2.98 - 3.00)=(2.98,2.98,3.00,3.02;0.8), ( ˜2.38 - 2.40)=(2.38,2.38,2.40,2.42;0.9), (˜2.34 - 2.40)=(2.34,2.34,2.40,2.46;0.7),(˜2.72 - 2.80)=(2.72,2.72,2.80,2.88;0.7),(2.19 - 2.20)=(2.19,2.19,2.20,2.21;0.8),(˜2.12 - 2.20)=(2.12,2.12,2.20,2.28;0.9),(˜102 -105)=(102,102,105,108;0.9)

## Results

For any value of λ in [0, 1], say λ =0.50 then the pareto Optimal solutions are as follows (table-2):

**Table 2: Optimal solutions (Pareto) of MMCP Model for λ =0.50**

| Variables & Objective Function | MMCP Model (without Entropy) | MMCP Model (with Entropy) |
|---|---|---|
| $z^*_1$ | 0.000000 | 1.285761 |
| $z^*_{21}$ | 0.000000 | 0.7854402 |
| $z^*_{31}$ | 19.20409 | 15.51402 |
| $z^*_{12}$ | 0.000000 | 0.09550524 |
| $z^*_{22}$ | 0.000000 | 1.129301 |
| $z^*_{32}$ | 23.33358 | 19.00942 |
| $z^*_{13}$ | 2.462324 | 1.871013 |
| $z^*_{23}$ | 0.000000 | 1.855958 |
| $z^*_{33}$ | 0.000000 | 3.453579 |
| P(z*) | 46.14557 | 45.68658 |
| W(z*) | 701.4866 | 707.1461 |
| L(z*) | 53.81290 | 56.08869 |
| E(z*) | ----------- | 1.469690 |

**Table 3: Optimal solutions(Pareto) of MMCP Model for different λ**

| Test | β* | E(z*) | P(z*) | W(z*) | L(z*) |
|---|---|---|---|---|---|
| Optimistic i.e. **λ** =1.0 | 0.5171632 | 1.466057 | 47.67087 | 720.8760 | 57.16139 |
| About Optimistic i.e. **λ** =0.80 | 0.5175277 | 1.468059 | 46.88355 | 715.3293 | 56.72653 |
| Moderate i.e. **λ** =0.50 | 0.5174324 | 1.469690 | 45.68658 | 707.1461 | 56.08869 |
| Pessimistic i.e. **λ** =0.0 | 0.5119525 | 1.463074 | 45.60090 | 694.3912 | 55.13732 |

## Discussion

The above table (Table 2) shows that the MMCP model without the entropy objective has most variables ($z^*_{11}$, $z^*_{21}$, $z^*_{12}$, $z^*_{22}$, $z^*_{23}$, $z^*_{33}$) equal to zero, whereas in the MMCP model with the entropy objective function, all variables take non-zero values. In this problem, the entropy objective function serves as a measure of diversification, disorder, or dispersal in farmland allocation, encouraging cultivation of all crop types across different seasons to meet market demand, even with only minor changes in the objectives (profit (P(z*)), water availability (W(z*)) and labour time (L(z*)). For a farmer aiming to distribute farmland across different crop types, the MMCP model with an entropy objective function offers a more realistic solution. Hence, the entropy-based model is more applicable in real scenarios, as it balances maximum profit and minimal water and labour requirements (with only very small changes) while ensuring maximum entropy in farmland distribution.

## Conclusion

This study presents various types of crop cultivation for agricultural management systems in different seasons. Here, the given article is to formulate the mathematical model for a crop production problem in a fuzzy environment with diversification (Entropy optimization) as an Entropy Maximization problem. Therefore, this entropy objective function acts as a measure of diversification/disorder/dispersal of farmland distribution to cultivate all types of crops in different seasons due to need of market demands with very small changes of objective values of profit, water availability and human resource (labour time). A nonlinear fuzzy decision-making method is employed to solve this real-life problem, specifically farm management for cultivating different crops. Like crop production planning and management problems, this method (Entropy optimization with fuzzy environments) can be applied to the problems of other fields such as Polyculture, Risk Analysis, Engineering Optimization, Environmental Analysis, etc.

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## Author Contributions

The sole author was responsible for the conceptualization, methodology, data collection, analysis, writing, and final approval of the manuscript.

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